Limiting current density in a crossed-field nanogap

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Using a mean-field theory, we have studied the quantum extension on the limiting current density in a crossed-field nanogap. When the gap spacing is less than the electron wavelength, our results show that the limiting current density is increased by a large factor from the classical values due to the effects of electron tunneling. The effects of the external magnetic field diminish with a decrease of gap spacing. Smooth transition from the classical regime to the quantum regime is demonstrated.

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The advent fields of nanoscience offer exciting capabilities in electronic, magnetic, mechanical, and biological systems [1]. Nanotechnology has shown great promise in flat panel displays [2,3], miniature coherent radiation sources [4], and nanodevices [5]. Miniature structures, such as nanotubes [5], nanogaps [6,7], and nanowires [8] ranging from sub-10 nm to 100s nm are readily fabricated [9]. In the nano scale, quantum effects will become important in the dynamics of beam-gap interaction. One important quantity that characterizes the beam-gap interaction is the limiting current that can be transmitted across a gap. The limiting current, which in the classical limit is known as the Child-Langmuir current [10] arises when the space charge in the gap creates a potential barrier that prohibits steady-state beam propagation. Using a mean-field theory, Lau *et al.* [11] has shown than this classical limiting current value may be increased by a large factor due to the effect of electron tunneling. In this paper, we extend Ref. [11] to include the effect of an external magnetic-field B, parallel to the diode surfaces. This extension is of fundamental interest because the magnetic fields have widely been used to control the electron flows across the gap. Note that the limiting current calculated here is independent of the emission mechanisms [12] of the electrons.

In the classical regime, an electron emitted from cathode is prohibited from reaching the anode when the magnetic field is larger than the Hull cutoff magnetic field [13] $B_H = \sqrt{2mV_g/eD^2 + (mu/eD)^2}$, where V_g is the dc gap voltage, D is the gap spacing, and u is the electron initial velocity. In this paper, we examine the quantum extension of the limiting current in a crossed-field gap when its gap spacing D is of order the electron wavelength, λ or less.

Consider the electrons with energy E being emitted from the cathode into a crossed-field gap with a gap spacing D, and with an external magnetic-field B parallel to the electrode surfaces. The anode is held at a dc voltage V_g with respect to a grounded cathode. From the mean-field theory [11], we solve the one-dimensional time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (m\Omega^2 x^2/2 - eV)\psi = E\psi, \tag{1}$$

Poisson equation

$$\frac{d^2V}{dx^2} = \frac{e\psi\psi^*}{\epsilon_o}, \tag{2}$$

and charge conservation relation

$$J = \frac{ie\hbar}{2m}\left(\psi\frac{d\psi^*}{dx} - \psi^*\frac{d\psi}{dx}\right), \tag{3}$$

in the gap region $0<x<D$. Here, ψ is the electron complex wave function, V is the mean space-charge field, J is the constant electron emission current density, and $\Omega = eB/m$ is the electron cyclotron frequency. Note that in deriving Eq. (1), we have omitted the consideration of the exclusion principle, and the intrinsic magnetic moment of the electrons. The self-magnetic field is also ignored in the deeply nonrelativistic regime treated here.

For convenience, we introduce the normalized parameters: $\overline{x} = x/D$, $\overline{J} = J/J_D$, $\epsilon = E/eV_D$, $\overline{V} = eV/E$, $\overline{n} = |\psi|^2/n_D$, $\phi_g = eV_g/E$ is the normalized gap voltage, $B/B_H = [(B/B_D)/\sqrt{\epsilon(1+\phi_g)}]$ is the magnetic field in units of Hull cutoff value $B_H = B_D\sqrt{\epsilon(1+\phi_g)}$. The normalized scales are the current scale $J_D = \epsilon_o\hbar^3/4em^2D^5$, the voltage scale $V_D = \hbar^2/2emD^2$, the density scale $n_D = \epsilon_o\hbar^2/2e^2mD^4$, and the magnetic-field scale $B_D = \hbar/eD^2$. Note that they only depend on the gap spacing D [14].

For a given gap spacing D, $\sqrt{\epsilon}$ measures the ratio of gap spacing D to the electron wavelength λ, and $\epsilon \gg 1$ is the classical limit. By using the normalized parameters, Eqs. (1)–(3) are rewritten into two coupled nonlinear equations of $p(\overline{x})$ and $\overline{V}(\overline{x})$:

$$\frac{1}{\epsilon}\frac{d^2p}{d\overline{x}^2} + [1 + \overline{V} - (1+\phi_g)(B/B_H)^2\overline{x}^2 - \alpha^2/p^4]p = 0, \tag{4}$$

and

$$\frac{d^2\overline{V}}{d\overline{x}^2} = p^2, \tag{5}$$

where

$$\alpha = \frac{1}{2}\,\frac{\overline{J}}{\epsilon^{3/2}}, \tag{6}$$

is a dimensionless *perveance*, which is proportional to the current density J. In deriving Eqs. (5) and (6), we have assumed that the complex wave function is of form $\psi = \sqrt{(n_D\epsilon)}\,p(\overline{x})\exp[i\theta(\overline{x})]$, where $p(\overline{x})$ and $\theta(\overline{x}) = \alpha\sqrt{\epsilon}\int_1^{x} d\overline{x}/p^2(\overline{x})$ are, respectively, the normalized real functions of the wave amplitude and phase, and $\theta(1)=\overline{k}$ is the phase at $\overline{x}=1$ (see below).

To obtain the boundary conditions for $p(\overline{x})$, we match the wave-function ψ at the anode ($\overline{x}=1$) to a transmitted plane wave $C\exp(i\overline{k}\overline{x})$, where $\overline{k}=\sqrt{\epsilon(1+\phi_g)}$ and $|C|^2 = n_D\overline{J}/2\overline{k}$ [from Eq. (3)]. The boundary conditions for Eqs. (4) and (5) become

$$p(1) = \sqrt{\alpha}/(1+\phi_g)^{1/4}, \tag{7a}$$

$$p'(1) = 0, \tag{7b}$$

$$\overline{V}(0) = 0, \tag{7c}$$

$$\overline{V}(1) = \phi_g, \tag{7d}$$

where the prime denotes the derivative with respect to $\overline{x}$. With the boundary conditions, we determine the limiting current density through the critical value of α, defined as α_q, so that for $\alpha > \alpha_q$, solutions to Eqs. (4) and (5) no longer exist.

In the classical limit at $\epsilon \gg 1$, we ignore the first term $\epsilon^{-1}(d^2p/d\overline{x}^2)$ in Eq. (4). Equation (5) becomes [15]

$$\frac{d^2\overline{V}}{d\overline{x}^2} = \frac{\alpha}{\sqrt{1+\overline{V}-(1+\phi_g)(B/B_H)^2\overline{x}^2}} \tag{8}$$

which is the governing equation for calculating the classical values of $\alpha_q(=\alpha_c)$ as a function of $B/B_H(<1)$ and ϕ_g, with boundary conditions: $\overline{V}(0)=0$ and $\overline{V}(1)=\phi_g$. Note that this classical limit of α_q is independent of ϵ (see Fig. 1 below). In the limit of no magnetic field, $B=0$, Eq. (8) gives the normalized classical Child-Langmuir current [10,11]

$$\alpha_{CL} = \frac{4}{9}(1+\sqrt{1+\phi_g})^3. \tag{9}$$

Figure 1 shows α_q as a function of ϵ for various values of magnetic-field B/B_H and normalized gap voltage ϕ_g. For a given B/B_H and ϕ_g, α_q increases with small values of ϵ (quantum regime), which clearly exceeds the classical limit at $\epsilon \gg 1$. This finding is due to tunneling of the electrons through the potential barrier provided by the mean space-charge field. The values of α_q at $\epsilon \gg 1$ are independent of ϵ, and they indeed equal to the classical values calculated by using Eq. (8). From Fig. 1, the transition from the classical regime to quantum regime occurs when $\epsilon = O(1)$, as expected (i.e., $\epsilon \approx 10$ for $\phi_g=1$, and $\epsilon \approx 3$ for $\phi_g=10$). In fact, the transition from the classical regime to the quantum regime depends explicitly on $\epsilon\phi_g \equiv V_g/V_D$, which measures the ratio of the applied gap voltage V_g to the voltage scale V_D. If we plot α_q as a function of $\epsilon\phi_g$, the transition occurs at about $\epsilon\phi_g = 10$ to 50 for $\phi_g=10$ to 100 and all $B/B_H < 1$. In the limit of $\epsilon \ll 1$, α_q scales as ϵ^{-1}, and is independent of B/B_H for a fixed ϕ_g. The last statement implies that the magnetic field can be ignored at very small gap spacing where the electron tunneling is dominant over the effect of magnetic field (see Fig. 2 below). This can also be seen from the dependence of x^2 in the magnetic-field term shown in Eq. (1).

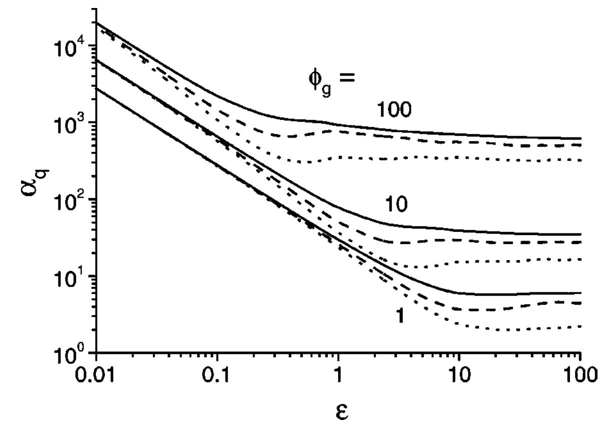

FIG. 1. The normalized limiting current α_q as a function of ϵ for various ϕ_g and B/B_H. Here, $\phi_g=1$, 10, and 100 (bottom to top), $B/B_H=0$ (solid lines), 0.7 (dashed lines), and 0.9 (dotted lines).

In Figs. 2 and 3, the solutions of the wave amplitude $p(\overline{x})$ and the mean space-charge field $\overline{V}(\overline{x})$ at $\alpha=\alpha_q$ and $\phi_g=1$ are plotted for various ϵ and B/B_H, where $B/B_H=0$ (solid lines) and $B/B_H=0.9$ (dashed lines). In the quantum regime at $\epsilon=1$, the electron tunneling effects are apparent as $1+\overline{V} < 0$ (i.e., $E+eV<0$) over a wide range of $\overline{x}$, and the effect of the magnetic field is negligible as the solutions are insensitive to B/B_H, as shown in Fig. 2. On the contrary, in

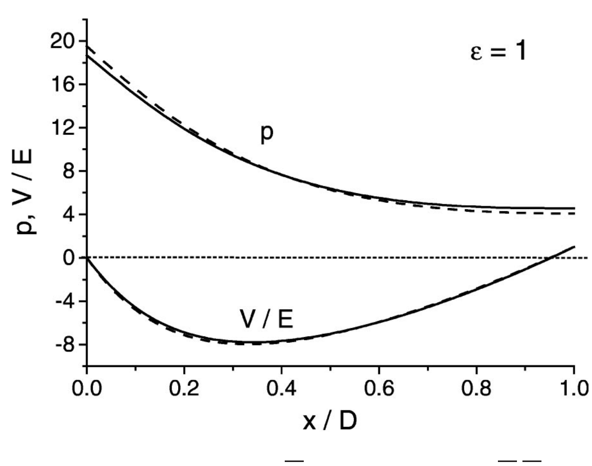

FIG. 2. The solutions of $p(\overline{x})$ from Eq. (4) and $\overline{V}(\overline{x}) = (V/E)$ from Eq. (5) at $\alpha=\alpha_q$, $\phi_g=1$, and $\epsilon=1$ (quantum regime) for $B/B_H=0$ (solid lines) and 0.9 (dashed lines).

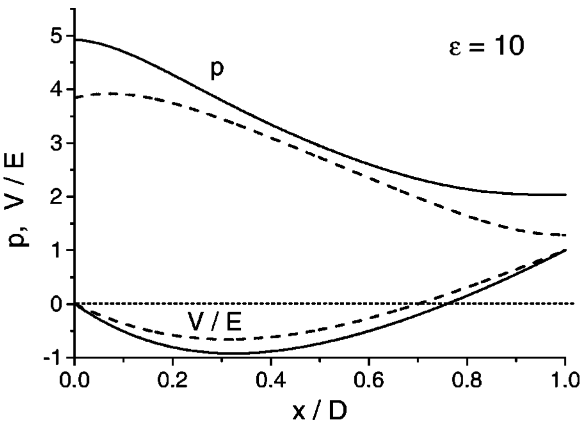

FIG. 3. The solutions of $p(\bar{x})$ from Eq. (4) and $\bar{V}(\bar{x}) = (V/E)$ from Eq. (5) at $\alpha = \alpha_q$, $\phi_g = 1$, and $\epsilon = 10$ (transition from the classical regime to quantum regime) for $B/B_H = 0$ (solid lines) and 0.9 (dashed lines).

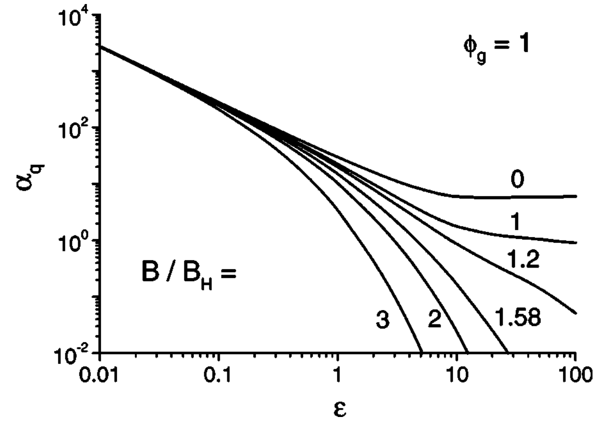

FIG. 4. The normalized limiting current α_q as a function of ϵ at $\phi_g = 1$ for various $B/B_H = 0$ to 3.

Fig. 3, at $\epsilon = 10$ where the transition from the classical regime to the quantum regime occurs (see Fig. 1), $1 + \bar{V} > 0$ (no electron tunneling), and the effect of the magnetic field is more significant.

As an example, assume $D = 20$ nm, and we have $V_D = 0.1$ mV, $J_D = 0.61$ A/cm^2, $n_D = 1.32 \times 10^{13}$ cm^{-3}, and $B_D = 1.65$ T. If we further set $\epsilon = 1$, $\phi_g = 1$, and $B/B_H = 0.9$, Fig. 1 gives $\alpha_q = 23.7$, whereas the classical value is $\alpha_c = 2.2$ from Eq. (8). The values of α_q and α_c show that the maximum current density that can be transmitted across a crossed-field nanogap with a gap spacing of 20 nm and a magnetic-field strength of about 2.1 T is 28.9 A/cm^2 from the quantum theory, and 2.68 A/cm^2 from the classical theory. The difference is more than a factor of 10. For comparison, at $B = 0$, $\alpha_q = 29.7$, and $\alpha_c = 6.25$ $(= \alpha_{cL})$ [from Eq. (9)], which is about a factor of 5 lower.

In the quantum regime, the gap is no longer magnetically insulated at $B > B_H$, as it is in the classical regime [13]. Due to the quantum effects, there is finite probability that the electrons will tunnel through the potential barrier to arrive at the anode. In Fig. 4, we show α_q as a function of ϵ for various values of magnetic-field $B/B_H = 0$ to 3 at $\phi_g = 1$. For $B > B_H$, we see that α_q remains finite, and it decreases with increase of ϵ.

In this study, we have assumed that the alignment of the external magnetic field is *perfectly* parallel to the electrode surfaces. The effect of the magnetic-field misalignments may be important especially when the gap spacing is small. In the classical regime [16], it is found that a small misalignment of the magnetic field can change the limiting current substantially for high magnetic field $(B > B_H)$, whereas the effect is less critical for low magnetic field $(B < B_H)$ [17]. Since the effect of magnetic field diminishes with the decrease of gap spacing as shown in the quantum regime given by this paper, small misalignments of the magnetic field may be negligible as long as $B < B_H$.

In summary, we have calculated the limiting current density of a crossed-field gap for a wide range of B/B_H, ϕ_g, and ϵ ranging from the classical regime to the quantum regime. From this formulation, the limiting current represents the maximum transmitted current that can reach the anode independent of the nature of the emission process at the cathode.

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